Vacuum Fluctuations induced Entanglement between Two Mesoscopic Systems

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We study the dynamics of a pair of molecular ensembles trapped inside a superconducting resonator through which they are strongly coupled via a microwave field mode. We find that entanglement can be generated via "vacuum fluctuations" even when the molecules and cavity field are initially prepared in their ground state. This entanglement is created in a relatively short time and without the need for further manipulation of the system. It does, therefore, provide a convenient scheme to entangle two mesoscopic systems, and may well be useful quantum information processing.

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Vacuum fluctuations can have important physical consequences, for example, in the Casimir effect [1] and Hawking radiation [2]. van der Waals interactions, i.e., attractive long-ranged interaction between neutral atoms or molecules, are also a kind of Casimir effect. It is an interesting question as to how vacuum fluctuations can be used to influence the properties of quantum entanglement between two systems. Quantum entanglement is a fundamental concept in quantum mechanics [3] and is also the physical resource in quantum information processing [4]. In fact, it has recently been shown possible to generate entanglement between a pair of particles via the vacuum modes of the radiation field [5, 6, 7].

In this paper, we study how vacuum fluctuations induce quantum entanglement between two mesoscopic systems, i.e., polar molecular ensembles [8] are placed inside a cavity, and strongly coupled by a single microwave mode. Recently, Rabl *et al.* [9] have proposed the realization of a quantum memory using such ensembles of polar molecules inside a superconducting resonator [10]. The energy difference between two internal states of a polar molecule is the order of GHz and polar molecules have significant electric dipole moments. A strong coupling to a microwave field via a transmission line can thus be achieved. In addition to the strength of the coupling, low-lying collective excitations can be coupled to the field and exploit the enhanced coupling to them, which scales as $\sqrt{N}$, where N is the number of molecules in the ensemble.

The dynamics of vacuum fluctuations [11] is hard to observe in ordinary systems. To show why this is so, we start the simple case of a two-level atom interacting with a quantized field. Conventionally, we use the Jaynes-Cummings model [12] in the interaction picture, Hamiltonian $H = g'[b^\dagger\sigma_- e^{i(\omega'-\omega_0')t} + \sigma_+ b e^{-i(\omega'-\omega_0')t}]$, to describe a two-level system $\sigma_\pm$ coupled to a quantized field b, for ω_0', ω' and g' are an energy difference between two-level atom, the frequency of the field and the Rabi frequency respectively. The rotating wave approximation (RWA) can usually be used because the two countering-rotating terms, $b\sigma_-$ and $\sigma_+ b^\dagger$, can be neglected; they carry a fast oscillation with the high frequency $\omega' + \omega_0'$. The RWA is, therefore, an excellent approximation for the optical frequency regime in the weak Rabi coupling limit. Clearly, this Hamiltonian will produce no evolution in the atoms and the photon field if they both start in the irrespective ground states. However, this approximation breaks down if the Rabi frequency g' is comparable to the frequencies ω' and ω_0'. In fact, the RWA is completely inadequate to describe the physical situation of a large number of molecules interacting with a microwave field in the strong coupling regime. It is thus necessary to go beyond RWA and, in essence, study the role of vacuum modes on the dynamics of the coupled atom-field system.

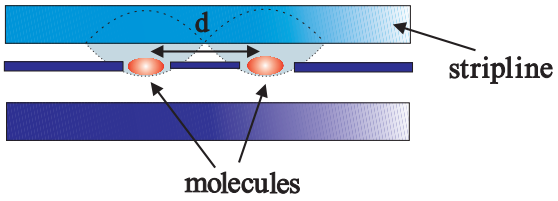

FIG. 1: Two molecular ensembles, separate with a distance d, are trapped inside a superconducting resonator and interact with a single-mode microwave field along the stripline.

We consider the case where molecules and the photon field are initially prepared in their ground states and show how the countering-rotating terms in the Hamiltonian do indeed induce quantum correlations between the molecules. The two ensembles of molecules exchange energy with the vacuum field due to the counter rotating terms. In physical terms we would say that this proceeds via virtual excitations of the cavity mode. We should bear in mind that these vacuum mode processes can be enhanced if collective excitations of the ensembles are used. As the dynamics takes place, the two molecular ensembles becomes entangled as the molecules are effectively coupled through the exchange of energy via the vacuum mode of the cavity. We will show that this entanglement can be generated in a comparatively short time. This result provides a novel route to entangle two mesoscopic systems. Relaxation and decoherence effects

should also be modest as the system is prepared in a vacuum state; a potentially crucial advantage for practical applications.

In this paper, we suppose the molecular ensembles are placed at a distance d apart in an effectively one-dimensional resonator as shown in Fig. 1. The two states, produced by the splitting of a rotational ground level by hyperfine interactions are chosen as the internal molecular states [9]. They have an energy difference ω, and are coupled to a single-mode microwave field with a frequency ω_0. Each of the molecule can be described by a spin-half particle $\vec{\sigma}_i$, and hence a collection of such spin-half particles can be described by a collective angular momentum operator $\vec{J} = \sum_{i=1}^{N} \vec{\sigma}_i$, when N is the number of molecules in one of the ensembles. The wavelength of microwave radiation will be much longer than the size of molecular ensemble. Hence, we can then assume the microwave field coupling to all molecules for the system of coupled molecules and radiation field in the form. We can now write down the Hamiltonian H ($\hbar = 1$) [13]:

$$H = \sum_{i=1}^{2} \omega_0 a^\dagger a + \omega J_{z_i} + g_i(a + a^\dagger)(J_{+_i} + J_{-_i}), \quad (1)$$

Here, $a^\dagger$ and a is the creation and annihilator operators of the cavity mode, J_{z_i} and J_{x_i} are the angular momentum operators to describe the collective inversion and transition for the i-th ensemble respectively, and $i = 1, 2$. The molecule-photon interaction strength is denoted by g_i for the i-th ensemble and they differ with a relative phase $\phi = 2\pi\omega d/c$ between the field and two ensembles, where c is the speed of the microwave photon. For simplicity, the magnitude for two Rabi coupling strengths are chosen to be the same, $|g_1| = |g_2|$, and $\phi \approx 0$. We consider the case where the molecules and photon field are in resonance, i.e. $\omega = \omega_0$, which is the optimal condition to observe the effect of the small vacuum fluctuations. We note that the Hamiltonian H has the same form as the Hamiltonian of the Dicke model without the rotating wave approximation [14]. The analysis we present here applies to thermal atomic ensembles as well as condensates [15]. Condensates would have the advantage of longer coherence times but also introduce nonlinear dynamical problems [16].

To study the quantum dynamics of this system, we need to find the eigen-states of the whole system. The Hilbert space of this system is extremely large as the number of molecules are correspondingly large. We can, however, consider the time evolution case where involves only the low-lying excitations of the molecules. We can make this approximation based on the Holstein-Primakoff transformation (HPT) [17] which enables us to express angular momentum operators in terms of harmonic oscillators. In this manner, we can take the leading approximation and map an angular momentum operator into a harmonic oscillators by taking the lowest order version of the HPT [18]. We then get $J_{x_i} \approx \sqrt{\omega N_i} x_{a_i}/\sqrt{2}$, $J_{y_i} \approx -\sqrt{N_i} p_{a_i}/\sqrt{2\omega}$, $J_{z_i} \approx (p_{a_i}^2 + \omega^2 x_{a_i}^2)/2\omega - N_i/2$ for x_{a_i} and p_{a_i} are the position and momentum operators, and N_i is the number of molecules in the cavity. This approximation is valid as long as $\langle p_{a_i}^2 + \omega^2 x_{a_i}^2(t)\rangle / \ll 2\omega N_i$ [18]. It should be a very good approximation for the number of molecules are sufficiently large [9].

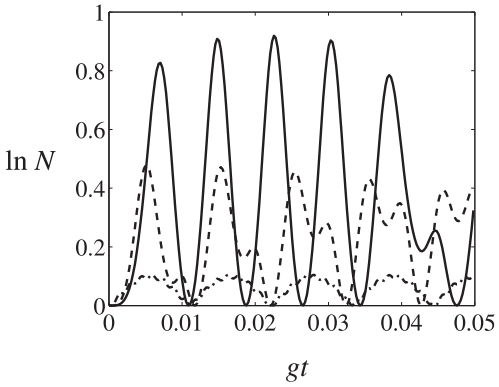

FIG. 2: Time evolution (the dimensionless time gt) of the logarithmic negativity $\ln N$ with the number of atoms N being 10^4. The different cases of $\omega = 300g$, $500g$ and $2000g$ are shown in solid, dashed and dash-dotted lines respectively.

For convenience, we write the cavity field operators in the phase-space representation: $x_c = (a^\dagger + a)/\sqrt{2\omega_0}$ and $p_c = i\sqrt{\omega_0}(a^\dagger - a)/\sqrt{2}$. We represent the system in terms of position and momentum operators. The Hamiltonian of system can then be rewritten in the form:

$$H' = \frac{1}{2}\sum_{i=1}^{2}(p_c^2 + p_{a_i}^2 + \omega_0^2 x_c^2 + \omega^2 x_{a_i}^2 + 4g_i\sqrt{N_i\omega\omega_0}\, x_{a_i} x_c). \quad (2)$$

We now want to find the dynamics as the ensembles interact and become entangled. This problem is clearly related to what of finding the entanglement between two harmonic oscillators in an open-ended harmonic chain [19].

This harmonic system will be in a Gaussian state, which allows us to quantify the general bipartite entanglement of the system. The density matrix of a Gaussian state can be completely determined by the second-order moments of the position and momentum operators of the system. We just need to study the reduced density matrix of the molecular part to find out the entanglement produced between the two ensembles. This reduced density matrix can be obtained by the tracing out the cavity mode. The reduced density matrix $\rho_{a_{1,2}}$ with matrix element $\langle X_i X_j + X_j X_i\rangle - 2\langle X_i\rangle\langle X_j\rangle$, where $X_i = x_{a_i}$ or p_{a_i}. A quantitative measure of entanglement can be obtained by using the logarithmic negativity [20] which gives us an upper bound for distillable entanglement. The logarithmic negativity in a Gaussian state can be found as

[20]

$$\ln N = -\sum_j \log_2[\min(1, |\gamma_j|)], \tag{3}$$

where γ_j are the symplectic eigenvalues of the matrix $\rho_{a_{1,2}}$.

We are now ready to investigate the entanglement dynamics of this system. We consider the initial state as the state of molecules and cavity, i.e., the state of the decoupled harmonic oscillators. In Fig. 2, we plot the time evolution of the entanglement between the ensembles. The system begins with a separable states and then the entanglement grows rapidly. In fact, the quantum state of two ensembles oscillates between being separable and entangled. This is very similar to the entanglement propagation in a harmonic chain in which the two oscillators are periodically entangled and unentangled [19].

Moreover, the system achieves the first maximal entanglement within the time interval $t^* = 5^{-3}g^{-1}$. We can estimate this time t^* with the realistic parameters. If we take g as 1 MHz [9], nearly maximal entanglement can be yielded within 5 ns. This means that a significant entanglement can be obtained rather quickly. Moreover, no further adjustment of the experimental parameters or making conditional measurements are required [21]. The time scale of this entanglement generation ($\sim$ 1 ns) is much shorter than the other decoherence sources such as inelastic collisions ($\sim$ 10 μs) and photon losses ($\sim$ 1 μs) [9]. Entanglement can therefore be observed before decoherence effect set in showing in a natural and efficient way to generate quantum entanglement for two mesoscopic systems. In addition, we can see that a larger ratio of g/ω can produce a larger degree of entanglement in Fig. 2, clearly indicating that counter-rotating terms cannot be neglected in this strong coupling limit.

We should note that thermal noise is the main potentially problem in entanglement production. It is of course impossible to prepare the perfect vacuum state of a molecular ensemble in an experiment due to finite temperature effects. We now assume these ensembles can be described as a thermal state with mean excitation number $\bar{n} = [\exp(\hbar\omega/k_BT) - 1]^{-1}$, for k_B is the Boltzmann constant and T is the temperature. We can estimate $\bar{n}$ to be of order 0.1 to 1 when $\omega \sim$1 GHz and $T \sim 1$ to 10 mK [9]. From this estimation, we can see that thermal effects cannot be neglected, and it is important to study their influence on entanglement. Time evolution of entanglement under the thermal effects is shown in Fig. 3. The amount of entanglement produced is shown to be lesser in the cases of higher $\bar{n}$. Moreover, the longer onset time of entanglement is required as shown in the higher temperature cases. But the entanglement can still be observed even if $\bar{n}$ is as high as 0.2. This result shows that a substantial amount of quantum entanglement can be effectively produced using thermal ensembles but colder molecules due result in a much better performance.

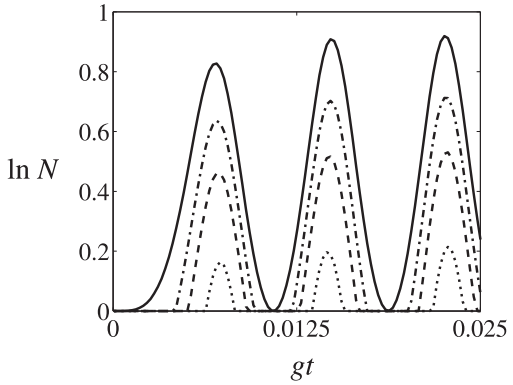

FIG. 3: The logarithmic negativity $\ln N$ is plotted against the dimensionless time gt with $\omega = 300g$ and $N = 10^4$. The solid, dash-dotted, dashed and dotted lines are represented $\bar{n} = 0$, 0.05, 0.1 and 0.2 respectively.

Having discussed the production of entanglement, we now study how to observe the quantum correlations. In this Gaussian system, the density matrix can be constructed if the uncertainties of these two ensembles can be obtained. This means that the entanglement of the two molecular ensembles can be determined just from the quantum uncertainties. In fact, non-resonant stimulated Raman scattering has been used to generate and verify the entanglement between two atomic ensembles [21, 22, 23]. In this scheme, the Stokes pulses are used to "write" the quantum information on the atomic ensembles and the scattered Stokes photons carry the information of excitations of each ensembles. Then, the two Stokes photon fields coming from each ensemble pass through a 50:50 beam splitter (BS) so that the two modes interfere and mix together. The conditional measurement of the resultant Stokes field can be preformed and entangle the two atomic ensembles [21, 22]. Similarly, the anti-Stokes pulses can be applied to read the excitations of the atoms and then the entanglement can be verified by measuring the correlations of photon fields.

We now adopt this method to determine the entanglement of the two molecular ensembles as shown in Fig. 4. We can apply two anti-Stokes pulses on these two ensembles (being vertical to the view of Fig. 1) by passing through two small pinholes of the cavity. These would have to be larger when the wavelength of the optical radiation used for the probe and therefore should not affect the quality of the microwave cavity. To read out the excitations of the molecules, we apply an anti-Stokes pulse to each ensemble which optically pump the excited states to a higher rotational states with a large detuning between the ground rotational state. In the Hesienberg picture, the output beam, after passing through the ensemble [22], is given by $a_i^O = \sqrt{\eta_i}c_i + \sqrt{1-\eta_i}a_i^I$, where a_i^O and a_i^I are the output field, the vacuum field operator and η_i is the effective transmission efficiency respectively. We can see

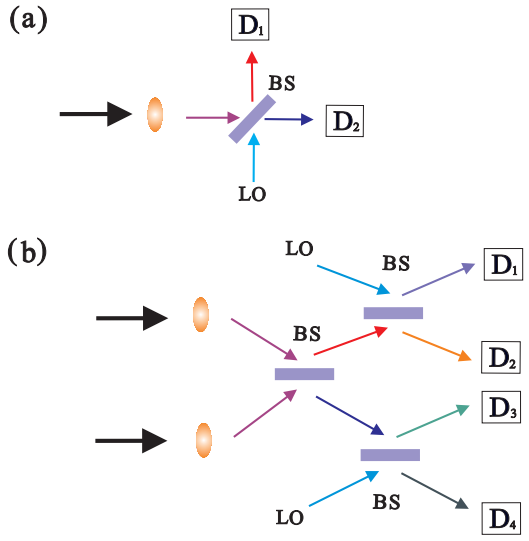

FIG. 4: In (a), Illustration for homedyne detection of measuring quantum correlations of a single molecular ensemble. Two anti-Stokes pump pulses are applied onto two molecular ensembles. The output field from molecules is superimposed on the field from a local oscillator (LO) at a 50:50 BS and then they are detected by the detectors D_1 and D_2 respectively. In (b), The output fields from two molecular ensembles are interfered with a 50:50 BS and then the fields are probed by the balanced homodyne detection.

that the output field directly carries the information of collective excitations of the molecules. Hence, the quantum state of the ensembles can be determined through the measurement of photon fields.

The measurement of the local and joint quantum correlations of molecules enables us to determine the logarithmic negativity. We then require the detection of individual ensemble and the two ensembles respectively. In Fig 4(a), we give a sketch of a scheme to measure the local quantum correlations of an individual ensemble by balanced homodyne detection method via inputting a local oscillator mode of a coherent state with a large amplitude and phase ϕ_l. [12]. The moments $\langle x_i^2\rangle$, $\langle p_i^2\rangle$ and $\langle x_i p_i + p_i x_i\rangle$ can all be probed by appropriately adjusting the phase angle ϕ_l. Similarly, the joint quantum correlations can also probed by this method [12]. This can be done by interfering two output fields with a 50:50 BS and then performing balanced homodyne detection as indicated in Fig. 4(b). The quadrature of the two modes can be thus determined.

In summary, we have found an efficient method to generate entanglement between two separate ensembles of molecules and proposed a method to measure it. We have assessed the role of the finite temperature on to the entanglement produced. It is useful to the quantum information processing with molecular systems in a superconducting device. Our study has implication of quantum optics of mesoscopic system in the strong coupling limit. We envisage that evaporative cooling of the trapped molecules will realized [24] so that the temperature can be lowered and the performance of quantum memory and entanglement generation be further improved.

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